# MICRO-WAVE MODULATION BY VARIABLE CIRCUIT ELEMENTS COMPRISING WAVE GUIDES OR CAVITY RESONATORS

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**ABSTRACT.** A driving mechanism similar to that of an electro-dynamic loudspeaker is utilised for varying the dimensions of a wave guide or cavity resonator according to the voice frequency signals so that micro-waves passing through the guide may either be amplitude or phase modulated and the micro-wave output of the cavity resonator may be amplitude modulated. The same mechanism is also applied for varying the length of a probe inside a guide resulting in load impedance modulation.

## INTRODUCTION

Amplitude modulation system wherein an impedance is varied in accordance with the signals to be transmitted is perhaps the first modulation system ever used in radio communication. In one of such system, the resistance of the antenna circuit is varied in accordance with the resistance of a carbon microphone. Another system uses the modulating voltage for varying the energy absorbed from the high frequency oscillations, a losser thermionic valve to the grid of which signal voltages are applied being connected across the tank circuit of a class C power amplifier. These modulating systems are rarely used at present. The micro-wave radio communication has been vastly developed during the last war and this paper describes methods of modulating micro-waves utilising principles stated above which were used in earliest days of radio. Micro-wave transmission circuits widely use wave guides and cavity resonators as transmission line and tuned circuits respectively. For any wave guide when the frequency is above cut-off, the phase velocity in the guide is greater than that in the free space ($v_p > c$) and the phase constant which is phase shift per unit length of the guide is equal to $\left(\frac{2\pi}{\lambda_g}\right)$ where $\lambda_g$ is the wave length inside the guide (Terman, 1943). Thus it is seen that by varying the length of a guide in accordance with a signal current it is possible to phase modulate the micro-waves propagating through a guide. The resonant frequency of a cavity resonator may be varied by changing its dimensions and if any dimension is varied in accordance with the signals to be transmitted the output of the resonator will correspondingly vary resulting in amplitude-modulated micro-waves when for maximum or minimum signal current, the resonator is tuned to the exciting micro-wave input. For a re-entrant coaxial resonator (Fig. 4) which we shall specially consider, the resonant wave length (Fink, 1947)

$$\lambda_0 = 2\pi \sqrt{\frac{2ha^2}{d}\left(\log_e \frac{b}{a}\right)}$$

and this paper describes means for varying $d$ in accordance with the signals. When load impedance modulation is desired the length of a probe inside the wave guide or a cavity resonator may be varied in accordance with the signal. This will also result in amplitude modulation. Variation of the radiation resistance of the exciting antenna inside of the wave guide will also result in amplitude modulation of the micro-waves.

## MODULATION INSIDE WAVE GUIDE

The cylindrical guides illustrated in figures 1 and 2 consist of two parts: $A$ and $B$. The part $A$ is fixed while the part $B$ is arranged to slide over $A$. The wall thickness of $B$ is very small in comparison with the diameter of the guide. In Fig. 1 the guide is excited in $TE_{1,1}$ mode while in Fig. 2

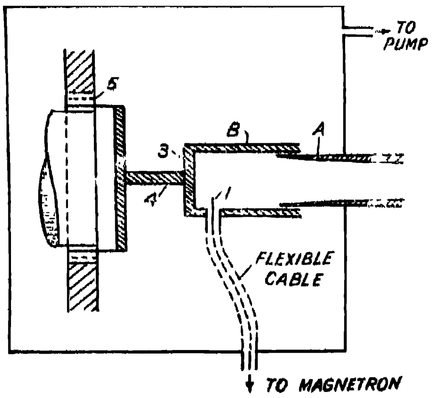


FIG. 1

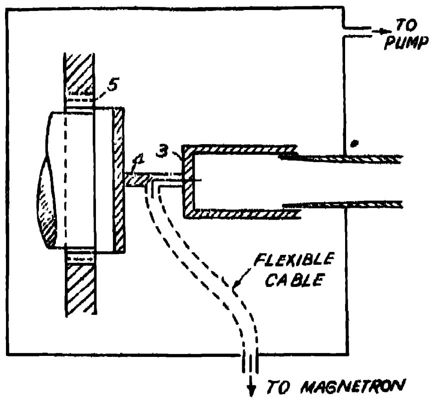


FIG. 2

the guide is excited in $TM_{01}$ mode. The metallic end piece 3 which is rigidly connected with $B$, is coupled through a rod 4 to a driving mechanism similar to that of a electrodynamic speaker. The centering arrangement of the voice coil 5 is not illustrated in the figure. The signal frequency amplifier feeds the moving coil wound over 2. Vibration of 2 causes linear movement of $B$ over $A$ resulting in variation in the length of the wave guide. Variation in length of the guide results in phase modulation of micro-waves emitted from the open end 6 of the guide. In Fig. 2 the coupling rod is partly hollow so that the central conductor of the coaxial cable exciting the antenna 1 may pass through it. The guide should be so mounted that lateral vibration of the same is prevented. The aerial 1 also should be prevented from vibrating.

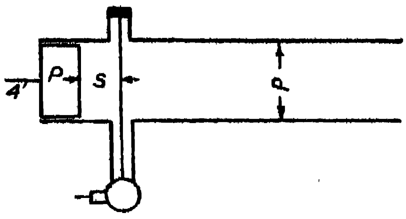


FIG. 3

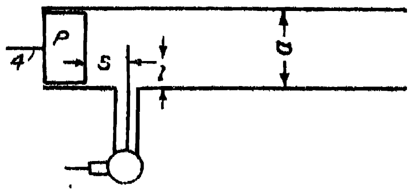


FIG. 4

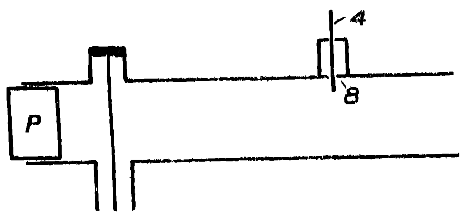


FIG. 5

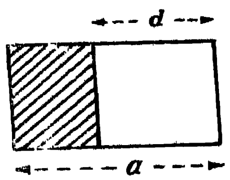


FIG. 5a

A method depending on the variation of radiation resistance of the exciting antenna 1 inside a rectangular wave guide 7 is illustrated in figures 3 and 4. The wave guide is excited in $TE_{0,1}$ mode. The radiation resistances of the antenna 1 of Figs. 3 and 4 are given by the expressions (M.I.T. Radar School, 1946)

$$R_r = 2Z_w K \sin^2\left(\frac{2\pi S}{\lambda_g}\right) \text{ for } K < 0.5 \qquad \ldots \quad (1)$$

$$\text{and } R_r = 2Z_w \frac{l^2}{ab} \sin^2\left(\frac{2\pi S}{\lambda_g}\right) \qquad \ldots \quad (2)$$

respectively, where

$K$ = ratio of the smaller to the larger dimension of the guide $= \left(\frac{a}{b}\right)$

wave impedance $Z_w = 120\,\pi \frac{\lambda_g}{\lambda_a}$ is the wave guide anologue of the characteristic impedance of conventional transmission lines,

$\lambda_g$ = wave length in the guide.

$\lambda_a$ = free space wave length of the source.

$S$ = distance of the plunger $P$ from the antenna 1.

$l$ = length of the antenna 1 (Fig. 4).

The net reactance of the antenna is a function of $S$ and $l$. The optimum condition of transmission (*i.e.*, when there is no reflection from the junction between the coaxial line and wave guide) both $l$ and $S$ are about $\lambda_g/6$. The plunger $P$ is coupled to a driving mechanism similar to that of figures 1 and 2, so that the distance $S$ is varied in accordance with the signal to be transmitted. When the arrangement is such that for maximum signal current, the optimum condition of transmission is reached, variation of $S$ will cause amplitude-modulation of the waves passing through the guide. The coupling arrangements, illustrated in figures 3 and 4, when properly adjusted for feeding power into the guide, are also well adjusted for taking power out. In the transmitting system shown in Fig. 7 which uses a biconical horn excited in $TEM$ mode, one portion $Q$ of the central conductor of the coaxial cable receives energy from the magnetron while another portion acts as the radiator. In this case, if the plunger $P$ is coupled to a driving mechanism described before and $S$ is varied according to the signal current, radiated waves will be amplitude modulated if for maximum or minimum signal current the optimum value of $S$ is reached. This system may be used for broadcasting purposes.

In Fig. 5, the length of a probe 8 coupled to the wave guide 7 at a suitable place, is varied according to the signals to be transmitted. The probe 8 must be in good electrical contact with the guide.

In each case the magnet $M$ and a part of the wave-guide should be enclosed inside a low pressure metal chamber which is continually evacuated. (This is not illustrated in figures 3-5.)

## MODULATION INSIDE CAVITY RESONATORS

The modes of cavity resonator may be designated by symbols $TE_{lmn}$ or $TM_{lmn}$ where $l$ is the number of half-period variations of the electric field along the length of the resonator or $X$-axis. In a cylindrical guide of length $x_0$ and radius $a$ in which $m$ represents the number of full period variations of the radial electric field along the direction and $n$ represents the number of half period variations of the angular component of the electric field in the radial $r$ direction for $TE_{lmn}$, the resonant frequencies which exist are obtained by (Sarbacher and Edson, 1946)

$$f_{lmn}=\sqrt{\left(\frac{cl}{2x_0}\right)^2+\left[\left(\frac{c}{2\pi}\right)\left(\frac{r'_{nm}}{a}\right)\right]^2}, \qquad l=1,\ 2,\ 3,\ldots$$

and for $TM_{lmn}$

$$f_{lmn}=\sqrt{\left(\frac{cl}{2x_0}\right)^2+\left[\left(\frac{c}{2\pi}\right)\left(\frac{r_{nm}}{a}\right)\right]^2}, \qquad l=0,\ 1,\ 2,\ 3,\ldots$$

where $r_{nm}$ and $r'_{nm}$ are the roots of the equations

$$J_n(r_{nm})=0$$
$$J'_n(r'_{nm})=0$$

Thus variation of $x_0$ causes change in resonant frequency for all modes other than $TM_{010}$
for which $l=0$ is permissible and

$$f_{010}=\frac{c}{2\pi}\left(\frac{2.405}{a}\right)$$

$$\lambda_{010}=2.61a$$

For $TM_{010}$ mode the resonant frequency depends only on the value of $a$, and the electric field is wholly parallel to the axis of the resonator and

$$E_x=AJ_0\left(r\frac{r_{01}}{a}\right)e^{j\omega t}$$

$$H_\phi=jA\frac{\omega\epsilon_1}{r_{01}}\,aJ_1\left(r\frac{r_{01}}{a}\right)e^{j\omega t}$$

The maximum energy $W_m$ stored in the magnetic field in this case is given by

$$W_m=\frac{\mu_1}{2}\int_{\phi=0}^{2\pi}\int_{\gamma=0}^{a}\int_{x=0}^{x_0}\left|H_\phi\right|^2 r\,d\phi\,dr\,dx$$

$$=\tfrac{1}{2}\left(\frac{a}{r_{01}}\right)^2 VA^2\epsilon_1\left(\frac{\omega}{c}\right)^2 J_1^2(r_{01})$$

$$=\tfrac{1}{2}VA^2\epsilon_1 J_1^2(r_{01})$$

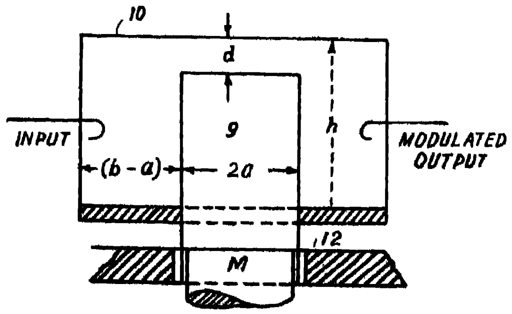


FIG. 6

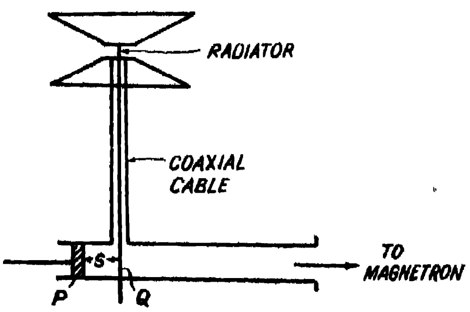


FIG. 7

From the equation of $W_m$ it is apparent that variation of $\epsilon_1$ by changing air pressure inside the resonator will cause variation of $(W_m/V)$ or the magnetic energy density.

The value of $x_0$ or $\epsilon_1$ may be varied by coupling the plunger $P$ (Fig. 8) to a driving mechanism similar to that previously described. If for maximum value of signal current $x_0=x_{res.}$, *i.e.*, resonating conditions are reached, the output from the resonator will be amplitude-modulated For $TM_{010}$ mode due to the variation of magnetic energy-density $(W_m/V)$ the output will be amplitude modulated but the percentage of modulation will be small.

In another transmitting system illustrated in Fig. 9 the resonator is coupled to a horn antenna through a slit $X$ and excited by a probe $E$.

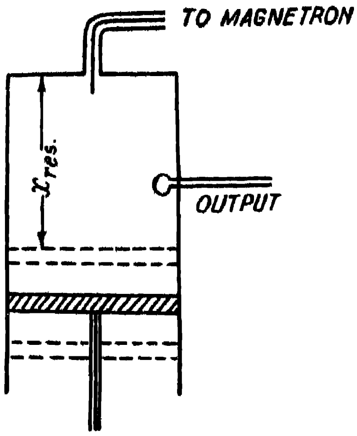


FIG. 8

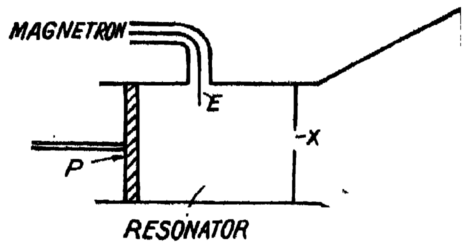


FIG. 9

Vibration of $P$ according to signal current will result in amplitude-modulation of the waves radiated by the horn if the resonator is tuned to the exciting waves either when the axial length of the resonator is maximum or when it is minimum in course of vibration.

In a further transmitting system, illustrated in Fig. 10, the resonator is coupled to a parabolic antenna and the plunger $P$ may be driven in a manner similar to that described before.

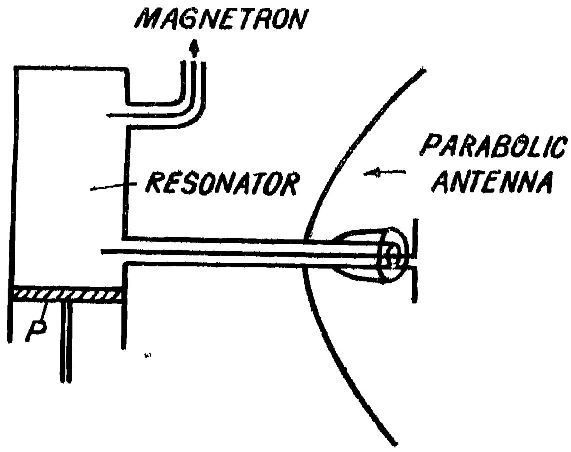


FIG. 10

In the re-entrant cavity resonator, illustrated in Fig. 6, the central post 9 slides inside the outer cylinder 10 so that $d$ may be varied. The bottom portion 11 of the outer cylinder is comparatively thicker so that the oscillation of 9 changes only the dimension $d$ of the resonator. The wall thickness of 9 is small and the moving coil 12 is wound over it. When signal power amplifier 5 feeds 12 the central post 9 will oscillate and the distance $d$ will vary according to the signals to be transmitted. Variation of $d$ causes change in the resonant wage length

$$\lambda_0 = 2\pi \sqrt{\frac{2ha^2}{d}\left(\log_e \frac{b}{a}\right)}$$

If in normal position of 9 (*i.e.* when no signals are applied to 12) the cavity resonator is not exactly tuned to the input micro-waves, variation of $d$ according to the signal current in 12 will cause variation in the output current taken through the loop and the radiated waves will be amplitude modulated, if resonating condition is approached when $d$ is maximum or minimum.

If the length of a losser probe inside the resonator is varied according to the signals to be transmitted, the load current will vary correspondingly and amplitude modulation results. In this case the resonator may be of any type like rectangular, cylindrical, spherical or the re-entrant type.

In each case the cavity resonator wholly or in part and the magnet $M$ should be enclosed in a low pressure metal chamber.

## DISCUSSIONS

In the amplitude modulating systems described above it is necessary to consider whether linear operation is obtained. Let us consider the system described with reference to Fig. 3. The magnetron feeds power to the wave

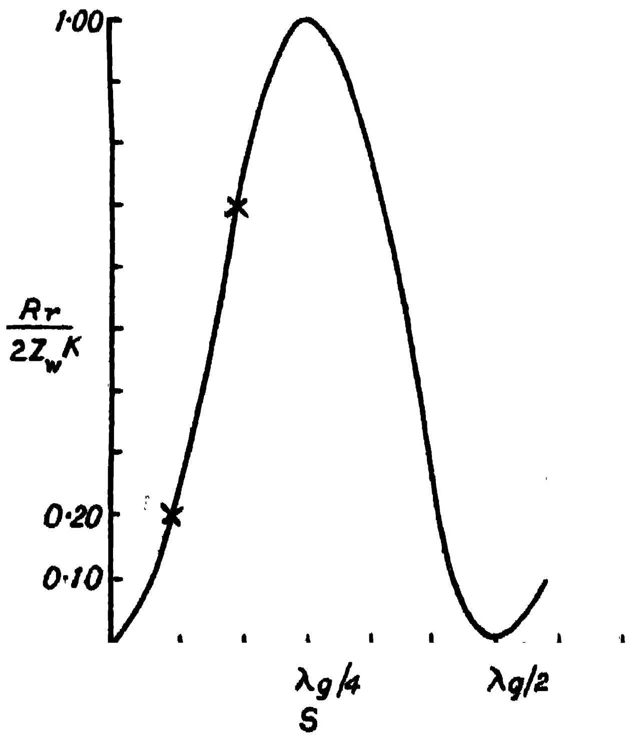


FIG. 11

guide through the coaxial line and for maximum possible transfer of power it is essential that the coaxial line is terminated in its characteristic impedance. The input impedance of the antenna is $(R_r + X_a)$ but by adjusting the reactance $X_s$ of the short circuited stub, the antenna reactance may be cancelled. Then the impedance looking into the wave guide will be merely the radiation resistance $R_r$. The variation or $R_r$ with respect to $S$ is shown in Fig. 11. The optimum value of $S$ is $\lambda_g/6$ and the variation in $R_r$ is substantially linear within the limits $S=\lambda_g/12$ and $S=\lambda_g/6$. If for no signal condition $S=\lambda_g/8$ and a limiter amplifier (as used in broadcast transmitters) is utilized so that the variation in $S$ never exceeds $\lambda_g/12$, the load impedance connected across the coaxial line will vary linearly and the modulation will be substantially linear. This system may be considered in principle similar to a load impedance modulation system. The performance of the system, described with reference to Fig. 4, appears to be similar to that of the system just considered so far as the linearity of modulation is concerned. The system described with reference to Fig. 5 which uses variation in the length of a probe is also a load impedance modulation system. The probe introduces a capacitive susceptance when it extends less than a free-space quarter wave-length into the guide but the variation in susceptance is not perfectly linear with the variation in the length of penetration of the probe. The arrangement should be such that maximum power flows through the guide when the probe length is zero and with no signal condition the probe length should be $(\lambda_a/8)$ where $\lambda_a$ is the free space wave length. In this case also a limiter amplifier is necessary in the signal input circuit because the variation in the length of the probe should not exceed $(\lambda_a/8)$. The modulation in this case is not satisfactory from the view point of linearity of modulation. If instead of using a probe inside the guide an asymmetrical inductive diaphragm of the type, shown in Fig. 5*b*, is used it is possible to vary the width $d$ of the open space which will result in the variation of the shunt susceptance $B$ which is given by the expression (M.I.T. Radiation Lab. Series, 1948)

$$B = -\frac{\lambda_g}{a} \cot^2 \frac{\pi d}{2a}\left(1 + \operatorname{cosec}^2 \frac{\pi d}{2a}\right).$$

If $(a/\lambda)$ is equal to (0.5) and $(d/a)$ does not exceed (0.7) the variation of $B$ with respect to change in $d$ will be substantially linear and faithful amplitude modulation is expected.

The modulation systems. described with reference to figures 6, 8, 9 and 10, which use cavity resonators are also cases of load impedance modulation. It is presumed that only one mode of oscillation exists in the resonator. The input impedance of the loaded cavity resonator is maximum when resonance occurs and when $d$ (Fig. 6) or $x_0$ (Fig. 8) is varied from its resonant value the input impedance rapidly drops to small fraction of the coupling loop or probe impedance. Near resonant frequency the input impedance of the cavity resonator varies in much the same manner as the impedance of a parallel resonant circuit and it appears that for small change in dimension of the cavity, substantially linear amplitude modulation will take place.

The methods of modulation described in this paper may be applicable to systems utilising several kilowatts of radio frequency power and the carrier wave length may be less than one centimetre.

In all the modulating arrangements comprising wave guides, the structure should be light as far as practicable. As for instance the portion $B$ including 3 (figures 1, 2) and the coupling arrangement of antenna 1 should be such that it is possible to make $B$ oscillate in the manner described. The low pressure chamber is used for reducing air vibration which is one of the potent damping factors.

When the wave length is less than 10 centimetres it is not feasible to produce amplitude modulated continuous waves by using valve modulating circuits. Signals may be transmitted at such high frequencies if time-modulated electric pulses are radiated.

From the expression of phase velocity $v_p$ inside a rectangular guide,

$$v_p = \frac{c}{\sqrt{1-(a/2b)^2}}$$

it appears that variation of $b$ (wider dimension of the guide) will cause variation in the phase velocity of the waves passing through the guide, resulting in phase modulation of the said waves. In this case due to the simultaneous variation of $Z_w$ which is equal to

$$\frac{120\pi}{\sqrt{1-(a/2b)^2}}$$

for any T. E. mode, amplitude modulation also takes place.

ACKNOWLEDGMENT

In conclusion the author wishes to express his thanks to Dewan Bahadur K. Rama Pai, M.A., Controller of Patents and Designs for his encouragement and kind pesmission to publish this paper.

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214, LOWER CIRCULAR ROAD,
CALCUTTA.

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## *ERRATUM*

In pages 10, 11 and 12 (No. I, Vol. 23), please read " $P_m$ in gms " in the place of " $P_m$ in gms per sq. cm " along the ordinates of the Figs. 2, 3 and 4.

**ACKNOWLEDGMENT**

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